

Resummation of rapidity logarithms in B meson wave functions

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ABSTRACT: We construct an evolution equation for the B meson wave functions in the k_T factorization theorem, whose solutions sum the double logarithms associated with the light-cone singularities, namely, the rapidity logarithms. The derivation is subtler than that of the Sudakov resummation for an energetic light hadron, due to the involvement of the effective heavy-quark field. The renormalization-group evolution in the factorization scale needs to be included in order to derive an ultraviolet-finite and scale-invariant kernel for resumming the rapidity logarithms. It is observed that this kernel is similar to that of the joint resummation for QCD processes in extreme kinematic regions, which combines the threshold and k_T resummations. We show that the resummation effect maintains the normalization of the B meson wave functions, and strengthens their convergent behavior at small spectator momentum. The resummation improved B meson wave functions are then employed in the leading-order analysis of the $B \rightarrow \pi$ transition form factors, which lead to approximately 25 % deduction in the large recoil region.

KEYWORDS: Summation of perturbation theory, Factorization, Decays of bottom mesons.

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1. INTRODUCTION

The B meson distribution amplitudes $\phi_B^\pm(k^+)$ are essential inputs for a perturbative analysis of exclusive B meson decays based on the collinear factorization theorem [1, 2, 3, 4, 5], where k^+ is the momentum carried by the light spectator quark. Their properties have been investigated intensively: models of ϕ_B^\pm with an exponential tail in the large k^+ region were proposed in [6]. Neglecting three-parton distribution amplitudes in a study based on equations of motion [7, 8], ϕ_B^\pm were found to contain step functions with a sharp drop at large k^+ [9]. The determination of the B meson distribution amplitudes from relevant data was discussed in [10, 11]. The asymptotic behavior of ϕ_B^+ was extracted from a renormalization-group (RG) evolution equation derived in the framework of the collinear factorization theorem, which decreases slower than $1/k^+$ [12]. That is, the B meson distribution amplitude is not normalizable, after taking into account the RG evolution effect. This feature was confirmed in a QCD sum rule analysis [13], which includes next-to-leading-order (NLO) perturbative corrections. It has been argued that a non-normalizable B meson distribution amplitude does not cause trouble in calculations of decay amplitudes, if the first inverse moment $\int dk^+ \phi_B^+(k^+)/k^+$ was involved [14, 15]. However, the non-normalizability does introduce difficulty in defining the B meson decay constant f_B via the integration of the B meson distribution amplitude [16].

It has been known that collinear factorization formulas for various exclusive B meson decay amplitudes, such as factorizable contributions and nonfactorizable contributions at higher powers, suffer end-point singularities [17]. These singularities imply that the k_T factorization theorem [18, 19, 20, 21, 22, 23] is a more appropriate theoretical framework for exclusive B meson decays [24]. Retaining parton transverse momenta k_T [25], as postulated in the perturbative QCD (PQCD) approach [26, 27, 28, 29] based on the k_T factorization

theorem, the end-point singularities disappear [30], and resultant predictions are in agreement with most of data [31, 32, 33]. It was then pointed out that the non-normalizability of ϕ_B^+ is also a consequence of the collinear factorization theorem [16]. Reanalyzing the RG evolution effect on the (unintegrated) B meson wave function in the k_T factorization theorem [16], it was found that the ultraviolet behavior of its evolution kernel is tamped. As a result, the RG evolution maintains the normalization of the B meson wave function.

As elaborated in [35], a k_T -dependent hadron wave function contains additional infrared divergences from the region with a loop momentum parallel to the Wilson line on the light cone. These light-cone singularities, cancelling each other in the collinear factorization theorem, appear in the k_T factorization theorem. To regularize the light-cone singularities, we have rotated the Wilson line from the light cone to an arbitrary direction n with $n^2 \neq 0$ [34, 35]. The higher-order wave function then depends on n^2 through the scale $\zeta_P^2 \equiv 4(P \cdot n)^2/n^2$, where P denotes the hadron momentum. The variation of ζ_P^2 introduces a factorization-scheme dependence of a hadron wave function. The evaluation of the NLO effective diagrams for the B meson and pion wave functions indicates the existence of the double logarithm $\ln^2 \zeta_P^2$ [36] and the single logarithm $\ln \zeta_P^2$ [37], respectively¹. These logarithms, with the same origin as the rapidity logarithms discussed in [34, 38, 39, 40], need to be resummed as $\ln \zeta_P^2$ becomes large. It is expected that the resummation of the rapidity logarithms in the B meson wave functions will reduce the scheme dependence.

In this paper we shall construct an evolution equation in the ratio $\zeta^2 \equiv \zeta_P^2/m_B^2 = 4(v \cdot n)^2/n^2$, $m_B(v)$ being the B meson mass (velocity), that resums the rapidity logarithms in the B meson wave functions. The idea is similar to that of the soft-collinear resummation for a light hadron wave function [34, 41]. The difference arises from the effective heavy-quark field involved in the definition of the former, which modifies the ultraviolet behavior of the evolution kernel. It will be shown that the RG evolution in the factorization scale μ_f , whose anomalous dimension also depends on ζ^2 , has to be taken into account in order to derive an ultraviolet finite kernel. That is, the evolutions in both ζ^2 and μ_f must be considered simultaneously for a consistent and complete treatment of the logarithmic corrections to the B meson wave functions, an observation in agreement with that in [39]. The solutions to the evolution equation contain the resummation of the rapidity logarithms, and their limits as $\zeta^2 \rightarrow \infty$ exist. It will be demonstrated that the wave function ϕ_B^- , which takes a finite value at $k^+ = 0$ before the resummation, vanishes after the resummation. The wave function ϕ_B^+ , which diminishes at $k^+ = 0$ before the resummation, approaches zero faster after the resummation. Namely, the effect from resumming the rapidity logarithms suppresses the behavior of the B meson wave functions near the end point $k^+ = 0$.

In Sec. II we construct the evolution equation that resums the rapidity logarithms in the B meson wave functions. The equation is then solved in the Mellin space in Sec. III, with the models of the B meson wave functions proposed in [9] as the initial condition of the evolution. As performing the inverse Mellin transformation of the above solutions, the extrapolation of the running coupling constant down to the low energy region is specified to avoid the Landau pole. It is observed that the two B meson wave functions, whose k^+

¹This logarithm corresponds to the scheme-dependent logarithm $\ln \nu'^2$ in [16], where a different definition for the B meson wave function was adopted.

dependencies differ dramatically before the resummation, become similar: both diminish faster than k^+ at $k^+ = 0$. The $B \rightarrow \pi$ transition form factors are then calculated for the B meson wave functions before and after the resummation. It is found that the resummation effect decreases the form factors by approximately 25 % at large recoil, which is attributed to the suppression of the B meson wave functions near the end point. We summarize our findings in Sec. IV, and collect the explicit expressions for the solutions of the evolution equation in Appendix A.

2. EVOLUTION EQUATION

The B meson wave functions Φ_B^\pm constructed in the k_T factorization theorem [18, 19, 20, 21, 22, 25],

$$\begin{aligned} & \langle 0 | \bar{q}(y) W_y(n)^\dagger I_{n;y,0} W_0(n) \Gamma h(0) | \bar{B}(v) \rangle \\ &= -\frac{if_B m_B}{4} \text{Tr} \left\{ \frac{1 + \not{v}}{2} \left[2 \Phi_B^+(t, y^2) + \frac{\Phi_B^-(t, y^2) - \Phi_B^+(t, y^2)}{t} \not{y} \right] \gamma_5 \Gamma \right\}, \end{aligned} \quad (2.1)$$

describe the distributions of the light parton in both the longitudinal direction denoted by $t = v \cdot y$ and the transverse direction denoted by y^2 . In the above definition $y = (0, y^-, \mathbf{y}_T)$ is the coordinate of the anti-quark field \bar{q} , h is the rescaled b quark field characterized by the B meson velocity v , and Γ represents a Dirac matrix. The Wilson line operator $W_y(n)$ is written as

$$W_y(n) = \mathcal{P} \exp \left[-ig \int_0^\infty d\lambda n \cdot A(y + \lambda n) \right], \quad (2.2)$$

where \mathcal{P} means the path-ordered integration, and g is the strong coupling constant. The vertical link $I_{n;y,0}$ at infinity does not contribute in the covariant gauge [42]. Note that a non-light-like vector n has been substituted for the null vector $n_- = (0, 1, \mathbf{0}_T)$, so that the light-cone divergences associated with the Wilson lines are regularized by $n^2 \neq 0$.

According to [36], the convolution of the NLO B meson wave functions with the leading-order (LO) hard kernel produces both the single and double logarithms of $\ln \zeta^2$, which become large as $(v \cdot n)^2 \gg n^2$. The latter is generated by the gluon exchange between the rescaled b quark and the Wilson line ending at the spectator coordinate y [36]. The double logarithm arises from the overlap of the collinear enhancement from a loop momentum l collimated to n and the soft enhancement from small l . To resum the above rapidity logarithms, we follow the strategy developed in [43]: we vary the velocity v under the constraint $v^2 = 1$, which respects the equation of motion for the rescaled b quark. Since the collinear dynamics is independent of v , the variational effect on the B meson wave function does not involve the collinear dynamics, and is factorizable. An evolution equation of the B meson wave function in v , or equivalently in ζ^2 , is then derived, whose solution resums the rapidity logarithms. As claimed in the Introduction, the resummation effect exists in the $\zeta^2 \rightarrow \infty$ limit, where the scheme dependence of the B meson wave functions is shown to disappear. The condition $v^2 = 1$ implies that the two components v^+ and v^- are not

independent, and that the derivative $dv^2/dv^+ = 0$ gives $dv^-/dv^+ = -v^-/v^+$. Therefore, the derivative of the wave function Φ_B should be understood as

$$v^+ \frac{d}{dv^+} \Phi_B = \left(v^+ \frac{\partial}{\partial v^+} - v^- \frac{\partial}{\partial v^-} \right) \Phi_B \equiv \epsilon_{\alpha\beta} v^\alpha \frac{\partial}{\partial v_\beta} \Phi_B, \quad (2.3)$$

with the anti-symmetric tensor $\epsilon_{\alpha\beta}$, $\epsilon_{+-} = -\epsilon_{-+} = 1$.

We have to differentiate the bare wave function $\Phi_B^{(b)}$ and the renormalized wave function Φ_B here, because they are defined in the heavy-quark effective theory. This differentiation is not necessary in the pion case as performing the Sudakov resummation, which is defined in full QCD. The difference becomes manifest as comparing their NLO expressions: the former contains a ζ^2 -dependent ultraviolet pole from the vertex correction formed by the rescaled b quark line and the Wilson line [36]

$$-\frac{\alpha_s C_F}{4\pi} \ln \zeta^2 \left(\frac{1}{\epsilon} + \ln \frac{4\pi\mu_f^2}{m_g^2 e^{\gamma_E}} \right), \quad (2.4)$$

m_g being an infrared regulator and γ_E being the Euler constant, but the latter does not. The variation of v is equivalent to that of ζ^2 ,

$$\frac{v \cdot n}{2\epsilon_{\alpha\beta} v^\alpha n^\beta} v^+ \frac{d}{dv^+} \Phi_B^{(b)}(x, k_T, \zeta^2, \mu_f) = \zeta^2 \frac{d}{d\zeta^2} \Phi_B^{(b)}(x, k_T, \zeta^2, \mu_f), \quad (2.5)$$

with the momentum fraction $x \equiv k^+/P^+$ of the spectator. Inserting $\Phi_B^{(b)} = Z_\Phi \Phi_B$ into Eq. (2.5), we have

$$\begin{aligned} \frac{1}{Z_\Phi} \zeta^2 \frac{d}{d\zeta^2} \Phi_B^{(b)}(x, k_T, \zeta^2, \mu_f) &= \frac{1}{Z_\Phi} \left(\zeta^2 \frac{d}{d\zeta^2} Z_\Phi \right) \Phi_B(x, k_T, \zeta^2, \mu_f) \\ &+ \zeta^2 \frac{d}{d\zeta^2} \Phi_B(x, k_T, \zeta^2, \mu_f), \end{aligned} \quad (2.6)$$

where Z_Φ is the ζ^2 -dependent renormalization constant of the B meson wave function resulting from Eq. (2.4).

The v dependence in $\Phi_B^{(b)}$ is introduced through the eikonal line associated with the rescaled b quark. Hence, the derivative with respect to v^+ applies to the Feynman rule for the rescaled b quark propagator

$$\frac{v \cdot n}{2\epsilon_{\alpha\beta} v^\alpha n^\beta} v^+ \frac{d}{dv^+} \frac{v^\mu}{v \cdot l} = \frac{\hat{v}^\mu}{v \cdot l}, \quad (2.7)$$

leading to the special vertex

$$\hat{v}^\mu \equiv \frac{v \cdot n}{2\epsilon_{\alpha\beta} v^\alpha n^\beta} \epsilon_{\rho\lambda} v^\rho \left(g^{\mu\lambda} - \frac{v^\mu l^\lambda}{v \cdot l} \right). \quad (2.8)$$

It is easy to see that the contributions from the two terms in the special vertex \hat{v}^μ , as contracted to a vertex in $\Phi_B^{(b)}$, cancel each other, when l is collimated to n . That is, \hat{v}^μ suppresses the dominant collinear dynamics associated with the Wilson lines. Therefore, the variational effect involves only the soft dynamics, and we can eikonalize the attachments

of the differentiated gluon emitted by the special vertex to all internal lines in the wave function. Following the reasoning in [43], the special vertex must appear at the outer most end of the eikonal line at leading-logarithm accuracy. Applying the Ward identity to the sum over all attachments of the differentiated gluon, we factorize the differentiated gluon out of $\Phi_B^{(b)}$.

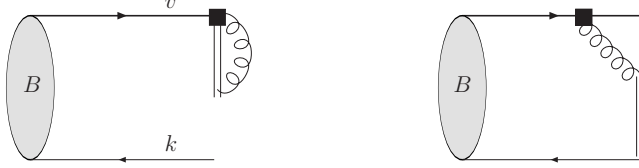


Figure 1: LO soft kernel for the ζ^2 -evolution equation, where the square at the end of the rescaled b -quark line represents the special vertex.

The derivative of $\Phi_B^{(b)}$ is then written as the convolution

$$\begin{aligned} \frac{1}{Z_\Phi} \zeta^2 \frac{d}{d\zeta^2} \Phi_B^{(b)}(x, k_T, \zeta^2, \mu_f) &= \frac{1}{Z_\Phi} K^{(b,1)} \otimes \Phi_B^{(b)}(x, k_T, \zeta^2, \mu_f) \\ &= K^{(b,1)} \otimes \Phi_B(x, k_T, \zeta^2, \mu_f), \end{aligned} \quad (2.9)$$

where the one-loop kernel $K^{(b,1)}$ collects the soft divergences generated by the differentiated gluons in Fig. 1. The two diagrams in Fig. 1 give $K^{(b,1)} = K_1^{(b,1)} + K_2^{(1)}$, with

$$K_1^{(b,1)} = -ig^2 C_F \int \frac{d^4 l}{(2\pi)^4} \frac{\hat{v} \cdot n}{(v \cdot l + i\epsilon)(l^2 + i\epsilon)(n \cdot l + i\epsilon)}, \quad (2.10)$$

$$\begin{aligned} K_2^{(1)} \otimes \Phi_B &= ig^2 C_F \int \frac{d^4 l}{(2\pi)^4} \frac{\hat{v} \cdot n}{(v \cdot l + i\epsilon)(l^2 + i\epsilon)(n \cdot l + i\epsilon)} \\ &\quad \times \Phi_B(x + l^+/P^+, k_T + l_T, \zeta^2, \mu_f). \end{aligned} \quad (2.11)$$

Note that the gluon exchange between the rescaled b quark and the spectator does not introduce a ζ^2 dependence [36]. It is obvious that the soft poles from $l \rightarrow 0$ cancel between $K_1^{(b,1)}$ and $K_2^{(1)}$, such that the evolution kernel is infrared finite. However, compared to the conventional resummation formalism for an energetic light hadron, Eq. (2.9) does not contain a hard kernel G . Once the heavy-quark expansion is implemented, the hard dynamics has been integrated out. The NLO corrections to the B meson wave function from gluons radiated by the spectator quark do not produce the double logarithm $\ln^2 \zeta$ either [36]. Then the ultraviolet divergence in $K_1^{(b,1)}$ seems to render the evolution kernel ill-defined. As stressed in the Introduction, this problem can be resolved by taking into account the ζ^2 -dependent RG evolution.

Substituting Eq. (2.9) into Eq. (2.6), we arrive at

$$\begin{aligned} \zeta^2 \frac{d}{d\zeta^2} \Phi_B(x, k_T, \zeta^2, \mu_f) &= K^{(b,1)} \otimes \Phi_B(x, k_T, \zeta^2, \mu_f) \\ &\quad - \frac{1}{Z_\Phi} \left(\zeta^2 \frac{d}{d\zeta^2} Z_\Phi \right) \Phi_B(x, k_T, \zeta^2, \mu_f), \end{aligned} \quad (2.12)$$

in which the second term plays the role of the counterterm for $K_1^{(b,1)}$. The cancellation of the ultraviolet poles is explicitly shown by computing $K_1^{(b,1)}$ and adopting Z_Φ in the $\overline{\text{MS}}$ scheme [16]

$$K_1^{(b,1)} = -\frac{\alpha_s C_F}{4\pi} \Gamma(\epsilon) \left(\frac{4\pi\mu_f^2}{\lambda^2} \right)^\epsilon \left(\frac{v \cdot n}{\epsilon_{\alpha\beta} v^\alpha n^\beta} \right)^2, \quad (2.13)$$

$$\begin{aligned} \delta K^{(1)} &\equiv \frac{1}{Z_\Phi} \zeta^2 \frac{d}{d\zeta^2} Z_\Phi \\ &= -\frac{\alpha_s C_F}{4\pi} \left[\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) \right] \left(\frac{v \cdot n}{\epsilon_{\alpha\beta} v^\alpha n^\beta} \right)^2, \end{aligned} \quad (2.14)$$

where λ is an infrared regulator and the term proportional to n^2 has been neglected relative to $v \cdot n$. The renormalization scale in $K_1^{(b,1)}$ has been set to the factorization scale μ_f , at which the B meson wave function is defined.

We apply the Mellin and Fourier transformations to the B meson wave function

$$\tilde{\Phi}_B(N, b, \zeta^2, \mu_f) = \int_0^1 dx (1-x)^{N-1} \int \frac{d^2 k_T}{(2\pi)^2} \exp(i\mathbf{k}_T \cdot \mathbf{b}) \Phi_B(x, k_T, \zeta^2, \mu_f), \quad (2.15)$$

under which the convolution in Eq. (2.11) reduces to

$$\int_0^1 dx (1-x)^{N-1} \int \frac{d^2 k_T}{(2\pi)^2} \exp(i\mathbf{k}_T \cdot \mathbf{b}) K_2^{(1)} \otimes \Phi_B = \tilde{K}_2^{(1)}(N, b, \zeta^2) \tilde{\Phi}_B(N, b, \zeta^2, \mu_f), \quad (2.16)$$

with the soft kernel

$$\begin{aligned} \tilde{K}_2^{(1)}(N, b, \zeta^2) &= ig^2 C_F \int \frac{d^4 l}{(2\pi)^4} \left(1 + \frac{l^+}{P^+} \right)^{N-1} \exp(-i\mathbf{l}_T \cdot \mathbf{b}) \frac{\hat{v} \cdot n}{(v \cdot l + i\epsilon)(l^2 + i\epsilon)(n \cdot l + i\epsilon)} \\ &= \frac{\alpha_s C_F}{2\pi} \left(\frac{v \cdot n}{\epsilon_{\alpha\beta} v^\alpha n^\beta} \right)^2 \left[K_0(\lambda b) - K_0 \left(\sqrt{\zeta^2} \frac{m_B b}{N} \right) \right], \end{aligned} \quad (2.17)$$

in the large ζ^2 limit, K_0 being the zero-order modified Bessel function of second kind. To simplify the integration, we have taken the equivalent limit $v^+ \rightarrow \infty$ with $n^+ = n^-$.

Equation (2.12) is rewritten, in the momentum (N) and coordinate (b) spaces, as

$$\zeta^2 \frac{d}{d\zeta^2} \tilde{\Phi}_B(N, b, \zeta^2, \mu_f) = \tilde{K}^{(1)}(N, b, \zeta^2, \mu_f) \tilde{\Phi}_B(N, b, \zeta^2, \mu_f), \quad (2.18)$$

with the renormalized soft kernel

$$\begin{aligned} \tilde{K}^{(1)}(N, b, \zeta^2, \mu_f) &= K_1^{(1)}(\mu_f) + \tilde{K}_2^{(1)}(N, b, \zeta^2) \\ &= -\frac{\alpha_s C_F}{2\pi} \left[\ln \frac{\mu_f b}{2} + \gamma_E + K_0 \left(\sqrt{\zeta^2} \frac{m_B b}{N} \right) \right], \end{aligned} \quad (2.19)$$

where $K_1^{(1)}$ is defined by $K_1^{(1)} \equiv K_1^{(b,1)} - \delta K^{(1)}$, and the infrared regulator λ has disappeared in the sum $K_1^{(1)} + \tilde{K}_2^{(1)}$. Equation (2.19) approaches $\ln b$ in the limit $\sqrt{\zeta^2} m_B b \gg N$, and $\ln N$ in the limit $N \gg \sqrt{\zeta^2} m_B b$. Namely, it involves the logarithms similar to those handled

in the joint resummation for QCD processes in extreme kinematic regions [44, 45, 46], which combines the threshold and k_T resummations. We mention that the ζ^2 -evolution equations derived in [47] differ from Eq. (2.18): the hard kernel G in their equation should not exist as explained before, once the rescaled b -quark field is adopted; they did not resum $\ln N$, so their equations are simpler; their equations for Φ_B^+ and Φ_B^- are different due to the incomplete B -meson light-cone projector adopted in their calculation, but we have confirmed that Φ_B^\pm obey the same equations; at last, they did not solve their equations, so the behavior of the solutions is not clear.

The relation $\Phi_B^{(b)} = Z_\Phi \Phi_B$ also leads to the RG equation

$$\mu_f \frac{d}{d\mu_f} \Phi_B = -\frac{1}{Z_\Phi} \mu_f \frac{dZ_\Phi}{d\mu_f} \Phi_B \equiv -\hat{\gamma}_B \Phi_B, \quad (2.20)$$

with the ζ^2 -dependent anomalous dimension

$$\hat{\gamma}_B = \frac{\alpha_s C_F}{2\pi} (\ln \zeta^2 - 2). \quad (2.21)$$

Solving Eq. (2.20) for arbitrary ζ^2 , we derive the RG evolution

$$\tilde{\Phi}_B(N, b, \zeta^2, \mu_f) = \exp \left[- \int_{\mu_0}^{\mu_f} \frac{d\mu}{\mu} \frac{\alpha_s(\mu)}{2\pi} C_F (\ln \zeta^2 - 2) \right] \tilde{\Phi}_B(N, b, \zeta^2, \mu_0), \quad (2.22)$$

with the initial scale μ_0 . Note that the RG evolution in the collinear factorization theorem drives the B meson distribution amplitude to converge slowly at large spectator momentum [12]. Obviously, the RG evolution in the k_T factorization behaves normally as indicated in Eq. (2.22).

Since the ultraviolet divergence and the light-cone divergence are from different kinematical region, the scheme and scale evolutions are commutable [39]. The commutativity of the derivatives with respect to ζ^2 and μ_f ,

$$\mu_f \frac{d}{d\mu_f} \zeta^2 \frac{d}{d\zeta^2} \Phi_B = \zeta^2 \frac{d}{d\zeta^2} \mu_f \frac{d}{d\mu_f} \Phi_B, \quad (2.23)$$

leads to the condition

$$\mu_f \frac{d}{d\mu_f} K^{(b,1)} = 0. \quad (2.24)$$

This is exactly the condition necessary for deriving the RG equation of the soft kernel,

$$\mu_f \frac{d}{d\mu_f} K^{(1)} = -\lambda_K, \quad (2.25)$$

with the anomalous dimension

$$\lambda_K \equiv \mu_f \frac{d}{d\mu_f} \delta K^{(1)} = \frac{\alpha_s C_F}{2\pi}. \quad (2.26)$$

The RG-improved soft kernel is then given by

$$\mathcal{K}^{(1)}(N, b, \zeta^2, \mu_f) = \tilde{K}^{(1)}(N, b, \zeta^2, \mu_c) - \int_{\mu_c}^{\mu_f} \frac{d\mu}{\mu} \lambda_K(\alpha_s(\mu)) \theta(\mu_f - \mu_c), \quad (2.27)$$

where the characteristic scale $\mu_c = a \sqrt{\zeta^2} m_B / N$, a being an order-unity constant, is chosen to remove large logarithms in the initial condition $\tilde{K}^{(1)}(N, b, \zeta^2, \mu_c)$. As demonstrated later, the suppression from the resummation on the B meson wave functions near $x = 0$ does not depend on a . The step function is introduced to terminate the evolution as $\mu_f < \mu_c$.

Solving Eq. (2.18) with the RG improved kernel in Eq. (2.27), one finds

$$\tilde{\Phi}_B(N, b, \zeta^2, \mu_f) = \exp \left[\int_{\zeta_0^2}^{\zeta^2} \frac{d\tilde{\zeta}^2}{\tilde{\zeta}^2} \mathcal{K}^{(1)}(N, b, \tilde{\zeta}^2, \mu_f) \right] \tilde{\Phi}_B(N, b, \zeta_0^2, \mu_f), \quad (2.28)$$

with the initial ratio ζ_0^2 . A reasonable value of the factorization scale μ_f is smaller than, and of order m_B [36]. We choose $\mu_f = a \zeta_0 m_B$, which will simplify the numerical analysis of the resummation effect in the next section. As a consequence, the upper bound of $\tilde{\zeta}^2$ is replaced by $N^2 \zeta_0^2$ in the limit $\zeta^2 \rightarrow \infty$ under the requirement $\mu_f > \mu_c$, and the scheme dependence of the B meson wave function disappears. One more advantage of the above choice of μ_f is that the effect from resumming the rapidity logarithms does not alter the normalization of the B meson wave function for arbitrary a : as $b \rightarrow 0$, which corresponds to the integration over k_T , and $N \rightarrow 1$, which corresponds to the integration over x , the upper bound $N^2 \zeta_0^2$ is identical to the lower bound ζ_0^2 , and the exponential in Eq. (2.28) becomes unity. We stress that the normalization of the B meson wave function remains unchanged for arbitrary ζ^2 as $a = 1$, which is the value of a we will work with in the next section. In this case the soft kernel in Eq. (2.27) vanishes: we have $\tilde{K}^{(1)}(1, 0, \zeta^2, \mu_c) = 0$, because the expansion of the Bessel function at small argument cancels the logarithm $\ln(\mu_c b)$ exactly, and the integral diminishes due to $\mu_c = \sqrt{\zeta^2} m_B \geq \mu_f = \zeta_0 m_B$.

For given μ_f , the combination of Eqs. (2.22) and (2.28) gives

$$\begin{aligned} \tilde{\Phi}_B(N, b) = \exp & \left[\int_{\zeta_0^2}^{N^2 \zeta_0^2} \frac{d\tilde{\zeta}^2}{\tilde{\zeta}^2} \mathcal{K}^{(1)}(N, b, \tilde{\zeta}^2, \mu_f) - \int_{\mu_0}^{\mu_f} \frac{d\mu}{\mu} \frac{\alpha_s(\mu)}{2\pi} C_F (\ln \zeta_0^2 - 2) \right] \\ & \times \tilde{\Phi}_B(N, b, \zeta_0^2, \mu_0), \end{aligned} \quad (2.29)$$

where the ζ^2 dependence has disappeared in the $\zeta^2 \rightarrow \infty$ limit. In this work we focus on the resummation of the double logarithms $\ln^2 \zeta^2$, and discuss its effect on the shape of the B meson wave function. For this purpose, we expand the Bessel function in Eq. (2.19), and the $\ln b$ term is absent. The evolution kernel then reads

$$\mathcal{K}^{(1)}(N, b, \zeta^2, \mu_f) = -\frac{\alpha_s(\mu_c)}{2\pi} C_F \ln a - \int_{\mu_c}^{\mu_f} \frac{d\mu}{\mu} \frac{\alpha_s(\mu)}{2\pi} C_F \theta(\mu_f - \mu_c). \quad (2.30)$$

We shall study the solution with the above simplified kernel numerically below.

3. RESUMMATION IMPROVED WAVE FUNCTIONS

In this section we perform the inverse Mellin transformation of Eq. (2.29) to obtain the x dependence of the resummation improved B meson wave functions

$$\Phi_B^\pm(x, k_T) = \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} (1-x)^{-N} \tilde{\Phi}_B^\pm(N, k_T), \quad (3.1)$$

where the constant c is chosen such that all poles of $\tilde{\Phi}_B^\pm(N, k_T)$ appear to the left of the contour in the N plane. Note that the exponential in Eq. (2.29) involves a branch cut on the negative real axis. We shall choose the contour which runs from minus infinity toward the origin below the branch cut, turns around at the origin along an infinitesimal circle, and then runs toward minus infinity above the branch cut. With the simplified kernel in Eq. (2.30), the Fourier transformation for the k_T dependence does not need to apply. We shall drop the RG evolution factor, which is irrelevant to the inverse Mellin transformation, and suppress the argument μ_0 . For the wave functions without the resummation, namely, the initial conditions of the ζ^2 evolution, we take the models proposed in [9] as an example. To make transparent the suppression mechanism near $x = 0$, we first consider the case with a fixed coupling constant. It will be explained how the exponential with the double rapidity logarithms diminishes the B meson wave function Φ_B^- at $x = 0$, which takes a finite value originally. We then investigate the case with a running coupling constant. Though the analysis becomes more complicated, the features of the resummation improved B meson wave functions remain.

3.1 Resummation with fixed α_s

Freezing the strong coupling constant α_s , the solution with the resummation of the rapidity logarithms reduces to

$$\tilde{\Phi}_B^\pm(N, k_T) = \exp \left[-\frac{\alpha_s C_F}{2\pi} \ln N \left(\ln a^2 + \ln N \right) \right] \tilde{\Phi}_B^\pm(N, k_T, \zeta_0^2), \quad (3.2)$$

where the integration over $\tilde{\zeta}^2$ has been worked out. We assume that the B meson wave function $\tilde{\Phi}_B^\pm(N, k_T, \zeta_0^2)$ possesses the factorized form

$$\Phi_B^\pm(x, k_T, \zeta_0^2) = \phi_B^\pm(x, \zeta_0^2) \phi(k_T), \quad (3.3)$$

with the models in [9]

$$\phi_B^-(x, \zeta_0^2) = \frac{2x_0 - x}{2x_0^2} \theta(2x_0 - x), \quad (3.4)$$

$$\phi_B^+(x, \zeta_0^2) = \frac{x}{2x_0^2} \theta(2x_0 - x). \quad (3.5)$$

The Mellin transformation of the above models give

$$\tilde{\Phi}_B^-(N, k_T, \zeta_0^2) = \frac{(1 - 2x_0)^{N+1} + 2x_0 N + 2x_0 - 1}{2x_0^2 N(N+1)} \phi(k_T), \quad (3.6)$$

$$\tilde{\Phi}_B^+(N, k_T, \zeta_0^2) = \frac{1 - (1 - 2x_0)^N (1 + 2x_0 N)}{2x_0^2 N(N+1)} \phi(k_T). \quad (3.7)$$

Performing the inverse Mellin transformation, we derive the resummation improved B meson wave function $\Phi_B^-(x, k_T) = \phi_B^-(x) \phi(k_T)$ with the distribution amplitude

$$\phi_B^-(x) = \frac{1-x}{2x_0^2} \exp \left(\frac{\alpha_s C_F}{2\pi} \right) \cos \left(\frac{\alpha_s C_F}{2} \ln a^2 \right) \theta(2x_0 - x)$$

$$\begin{aligned}
& -\frac{1}{2x_0^2} \int_{-\infty}^{+\infty} \frac{dt}{\pi} \frac{(1-x)^{e^t}}{1-e^t} \left[(1-2x_0)^{1-e^t} \theta(x-2x_0) - 2x_0 e^t + 2x_0 - 1 \right] \\
& \times \exp \left[-\frac{\alpha_s C_F}{2\pi} (t^2 + t \ln a^2 - \pi^2) \right] \sin[\alpha_s C_F (t + \ln a)], \tag{3.8}
\end{aligned}$$

in the momentum fraction space. The first term in the above expression arises from the $N = -1$ pole, and the second term containing an integral arises from the discontinuity of the integrand along the branching cut on the negative real axis. The variable change $N = \exp(t+i\pi)$ ($N = \exp(t-i\pi)$) has been applied to the contour above (below) the branch cut. The exponential with the exponent $-\ln^2 N$ in Eq. (3.2) suppresses the residue of the $N = 0$ pole, and diminishes the B meson wave function at $x = 0$ [48]. It is then easy to realize that this result is not modified by variation of a , because of $\ln^2 N \gg \ln a^2 \ln N$. The step function $\theta(x - 2x_0)$ in the square bracket is attributed to the vanishing of the inverse Mellin transformation of $(1-2x_0)^{N+1}/[N(N+1)]$ as $x < 2x_0$: in this case we must close the contour in the N plane through the positive real axis, such that the semicircle at infinity does not contribute. Hence, this contour does not enclose any poles, and the considered inverse Mellin transformation gives a null value. The x dependence of the resummation improved $\phi_B^-(x)$ with the example set of parameters $\alpha_s = 0.3$, $\zeta_0 = e/10$ and $a = 1$ is displayed in Fig. 2: $\phi_B^-(x)$ indeed vanishes at $x = 0$, and becomes smooth with a quick descending at large x , even though the initial condition contains a step function. It is trivial to verify that the normalization condition $\int_0^1 dx \phi_B^-(x) = 1$ is respected, albeit with the negative value of $\phi_B^-(x)$ at large x . Moreover, we find that the resummation improved $\phi_B^-(x)$ decreases faster than x at small x , $d\phi_B^-(x)/dx|_{x=0} = 0$.

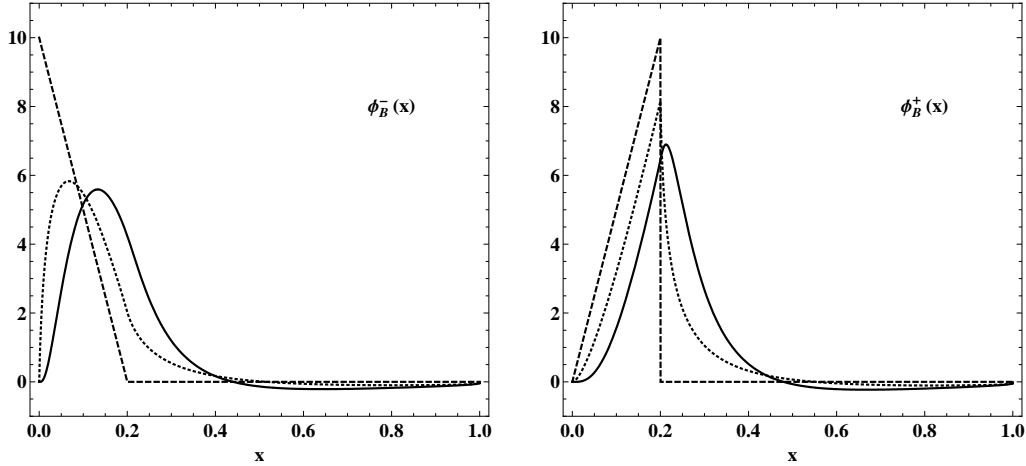


Figure 2: x dependence of the B meson distribution amplitudes. The dashed, dotted and solid curves correspond to the initial condition $\phi_B^\pm(x, \zeta_0^2)$, and the resummation improved $\phi_B^\pm(x)$ for fixed $\alpha_s = 0.3$ and for running α_s with $\zeta_0 = e/10$ and $a = 1$, respectively.

Following the same procedure, we derive the resummation improved wave function

$\Phi_B^+(x, k_T) = \phi_B^+(x)\phi(k_T)$ with the distribution amplitude

$$\begin{aligned}\phi_B^+(x) = & -\frac{1-x}{2x_0^2} \exp\left(\frac{\alpha_s C_F}{2\pi}\right) \cos\left(\frac{\alpha_s C_F}{2} \ln a^2\right) \theta(2x_0 - x) \\ & -\frac{1}{2x_0^2} \int_{-\infty}^{+\infty} \frac{dt}{\pi} \frac{(1-x)e^t}{1-e^t} \left[1 - (1-2x_0)^{-e^t} (1-2x_0 e^t) \theta(x-2x_0)\right] \\ & \times \exp\left[-\frac{\alpha_s C_F}{2\pi} (t^2 + t \ln a^2 - \pi^2)\right] \sin[\alpha_s C_F(t + \ln a)].\end{aligned}\quad (3.9)$$

A similar reason leads to the observation that $\phi_B^+(x)$ and its first derivative vanish at $x = 0$, i.e., $\phi_B^+(0) = 0$ and $d\phi_B^+(x)/dx|_{x=0} = 0$, which is not revised by variation of a . Namely, the resummation improved $\phi_B^+(x)$ decreases faster than the initial condition in Eq. (3.5) as x approaches zero. The shape of $\phi_B^+(x)$ with $\alpha_s = 0.3$, $\zeta_0 = e/10$ and $a = 1$ is displayed in Fig 2, which also becomes smooth with a quick descending at large x . We have checked that the normalization $\int_0^1 dx \phi_B^+(x) = 1$ is not changed by the resummation effect. It is worth mentioning that the model of the B meson wave function proposed in [28], which decreases like x^2 at small x and exhibits a Gaussian tail at large x , agrees with the above features.

3.2 Resummation with running α_s

We now investigate the resummation effect with a running coupling constant α_s . To avoid the Landau singularity, we follow the analytic parametrization proposed in [49]

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0} \left[\frac{1}{\ln(\mu^2/\Lambda_{\text{QCD}}^2)} - \frac{\Lambda_{\text{QCD}}^2}{\mu^2 - \Lambda_{\text{QCD}}^2} \right], \quad (3.10)$$

to one-loop level, where Λ_{QCD} is the QCD scale, and $\beta_0 = (11N_c - 2N_f)/3$ is the first coefficient of the β function, with N_c and N_f being the numbers of colors and flavors, respectively. Concentrating only on the resummation effect, we derive the following solution from Eq. (2.29)

$$\tilde{\Phi}_B^\pm(N, b) = \exp\left[-\frac{2C_F}{\beta_0}(A_1 \ln a + B_1)\right] \tilde{\Phi}_B^\pm(N, b, \zeta_0^2), \quad (3.11)$$

where the functions A_1 and B_1 are

$$\begin{aligned}A_1 = & \ln \frac{\hat{\mu}_f}{\hat{\mu}_f - \ln N} + \ln \frac{\mu_f^2 - N^2 \Lambda_{\text{QCD}}^2}{\mu_f^2 - \Lambda_{\text{QCD}}^2}, \\ B_1 = & \frac{1}{2} \ln^2 N + (\hat{\mu}_f - \ln N) \ln(\hat{\mu}_f - \ln N) - \hat{\mu}_f \ln \hat{\mu}_f + \ln N \left(\ln \frac{\mu_f \Lambda_{\text{QCD}}}{\mu_f^2 - \Lambda_{\text{QCD}}^2} + \ln \hat{\mu}_f + 1 \right) \\ & - \text{Li}_2\left(-\frac{\mu_f}{\Lambda_{\text{QCD}}}\right) + \text{Li}_2\left(-\frac{\mu_f}{N \Lambda_{\text{QCD}}}\right) - \text{Li}_2\left(-\frac{N \Lambda_{\text{QCD}}}{\mu_f}\right) + \text{Li}_2\left(-\frac{\Lambda_{\text{QCD}}}{\mu_f}\right),\end{aligned}\quad (3.12)$$

with $\hat{\mu}_f = \ln(\mu_f/\Lambda_{\text{QCD}})$.

Employing the models in Eqs. (3.4) and (3.5) for the initial conditions, and performing the inverse Mellin transformation of $\tilde{\Phi}_B^\pm(N, k_T)$, we finally arrive at the distribution amplitudes

$$\begin{aligned} \phi_B^-(x) = & \frac{1-x}{2x_0^2} \exp \left[-\frac{2C_F}{\beta_0}(A_2 \ln a + B_2) \right] \cos \left[-\frac{2C_F}{\beta_0}(A_3 \ln a + B_3) \right] \theta(2x_0 - x) \\ & - \frac{1}{2x_0^2} \int_{-\infty}^{+\infty} \frac{dt}{\pi} \frac{(1-x)e^t}{1-e^t} \left[(1-2x_0)^{1-e^t} \theta(x-2x_0) - 2x_0 e^t + 2x_0 - 1 \right] \\ & \times \exp \left[-\frac{2C_F}{\beta_0}(A_4 \ln a + B_4) \right] \sin \left[-\frac{2C_F}{\beta_0}(A_5 \ln a + B_5) \right], \end{aligned} \quad (3.13)$$

$$\begin{aligned} \phi_B^+(x) = & -\frac{1-x}{2x_0^2} \exp \left[-\frac{2C_F}{\beta_0}(A_2 \ln a + B_2) \right] \cos \left[-\frac{2C_F}{\beta_0}(A_3 \ln a + B_3) \right] \theta(2x_0 - x) \\ & - \frac{1}{2x_0^2} \int_{-\infty}^{+\infty} \frac{dt}{\pi} \frac{(1-x)e^t}{1-e^t} \left[1 - (1-2x_0)^{-e^t} (1-2x_0 e^t) \theta(x-2x_0) \right] \\ & \times \exp \left[-\frac{2C_F}{\beta_0}(A_4 \ln a + B_4) \right] \sin \left[-\frac{2C_F}{\beta_0}(A_5 \ln a + B_5) \right], \end{aligned} \quad (3.14)$$

where the functions A_i and B_i ($i = 2-5$) are collected in Appendix A. Their x dependencies are also shown in Fig. 2, which do not differ much from the curves for the fixed α_s , but with the peak positions shifting toward large x a bit. The normalization condition and the suppression at $x = 0$, $\phi_B^\pm(0) = 0$ and $d\phi_B^\pm(x)/dx|_{x=0} = 0$, are maintained. We mention that the ‘‘Wandzura-Wilczek relations’’ [50] of the two B meson wave functions do not hold anymore after including the resummation of the rapidity logarithms. This observation is not unexpected, since it is not clear that such relations sustain under radiative corrections to all orders [51, 52].

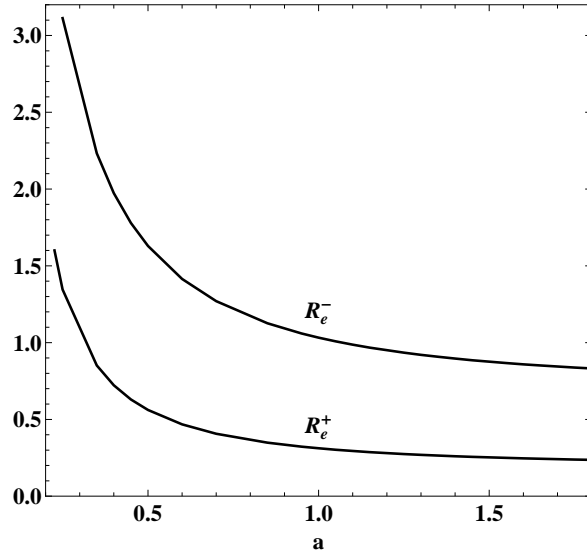


Figure 3: a dependence of the ratios R_e^\pm for $\zeta_0 = e/10$.

To illustrate the a dependence of the resummation effect, we consider the ratios of the resummation improved B meson distribution amplitudes over the initial conditions for $x = 0.1$,

$$R_e^\pm = \frac{\phi_B^\pm(0.1)}{\phi_B^\pm(0.1, \zeta_0^2)}. \quad (3.15)$$

These ratios for $\zeta_0 = e/10$ are displayed in Fig. 3, which decrease with a and become stable as $a > 1$. It implies that the peaks of $\phi_B^\pm(x)$ will be pushed toward large x , as a increases. However, as long as a remains of order unity, namely, away from zero, the shape change will not be significant. In this case the characteristic scale $\mu_c \sim O(\sqrt{\zeta^2} m_B/N)$ removes the large logarithm in the initial condition $K^{(1)}(N, b, \zeta^2, \mu_c)$, and the kernel governing the resummation effect roughly stays the same.

A hard kernel H , defined as the difference between the QCD quark diagrams and the effective diagrams, also contains the rapidity logarithms. In principle, these logarithms should be resummed by solving the evolution equation

$$\zeta^2 \frac{d}{d\zeta^2} \tilde{H}(N, b, \zeta^2, \mu_f) = -\tilde{K}_H^{(1)}(N, b, \zeta^2, \mu_f) \tilde{H}(N, b, \zeta^2, \mu_f), \quad (3.16)$$

which arises from the ζ^2 independence of the QCD quark diagrams. After treating $\ln \zeta^2$ in both the B meson wave functions and the hard kernel, the scheme dependence disappears completely. Strictly speaking, the ζ^2 dependence in the pion wave functions should be treated too. The above subjects will be studied in a separate work. Here we investigate the resummation effect on the $B \rightarrow \pi$ transition form factors involved in semileptonic decays [36]. For this purpose, it is legitimate to simply convolute the LO hard kernel with the B meson wave functions before and after the resummation. We take this opportunity to correct a typo in Eq. (35) of [36], where the $+(1/2) \ln(\zeta_1^2/m_B^2)$ term should be replaced by $-(1/2) \ln(\zeta_1^2/m_B^2)$. Correspondingly, the term $-(1/2) \ln(m_B^2/\zeta_1^2) [3 \ln(m_B^2/\zeta_1^2) + 2]$ in Eqs. (54) and (56) of [36] needs to be changed into $-(1/2) \ln(m_B^2/\zeta_1^2) [\ln(m_B^2/\zeta_1^2) + 2]$, and the term $+(1/2) \ln(m_B^2/\zeta_1^2) [3 \ln(m_B^2/\zeta_1^2) + 2]$ involved in the coefficient c_1 in Eq. (61) should be $+(1/2) \ln(m_B^2/\zeta_1^2) [\ln(m_B^2/\zeta_1^2) + 2]$. The $B \rightarrow \pi$ form factors $f_{B \rightarrow \pi}^+(q^2)$ and $f_{B \rightarrow \pi}^0(q^2)$ on the lepton-pair invariant mass q^2 in the k_T factorization approach are presented in Fig. 4, where the RG evolution effect in Eq. (2.29) has been included. It is observed that the resummation effect decreases both form factors by about 25% at $q^2 = 0$, which is attributed to the suppression of the end-point behavior of the B meson wave functions.

4. Conclusion

In this paper we have constructed the evolution equation for the B meson wave functions in the k_T factorization theorem, whose solution sums the rapidity logarithms from the light-cone singularities. Since the B meson wave functions are defined with the rescaled b -quark field, the ultraviolet behavior of the evolution kernel differs from that in conventional resummations. It has been shown that the RG evolution in the factorization scale must be taken into account simultaneously, in order to have a well-defined evolution kernel.

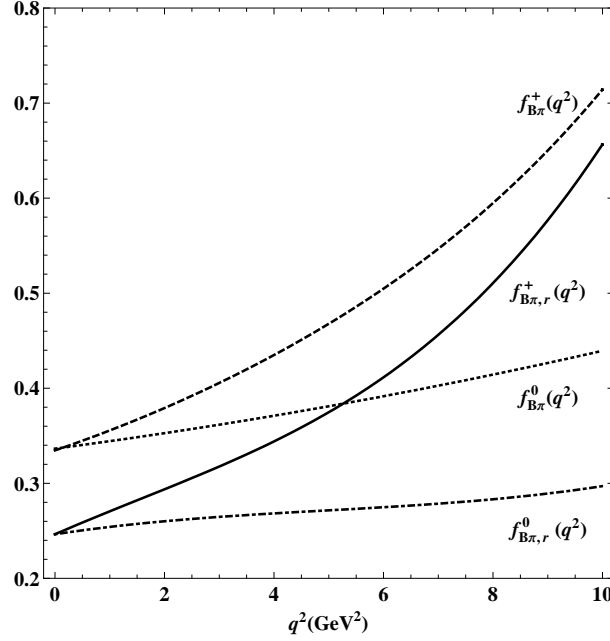


Figure 4: q^2 dependence of the $B \rightarrow \pi$ form factors $f_{B \rightarrow \pi}^+(q^2)$ and $f_{B \rightarrow \pi}^0(q^2)$ in the k_T factorization with the resummation effect (solid and dash-dotted lines, respectively) and without the resummation effect (dashed and dotted lines, respectively).

The commutativity between the scheme and RG evolutions demands the RG invariance of the evolution kernel, which is the key step to perform the resummation. It is interesting to notice that this new resummation formalism is similar to the joint resummation, which combines the threshold and k_T resummations. We have demonstrated that the resummation effect respects the normalization of the B meson wave functions, and strengthens their convergent behavior near the end point of the spectator momentum. These features cannot be obtained in the collinear factorization theorem, in which the B meson distribution amplitude loses the normalizability under the RG evolution. It has been found that the resummation improved B meson wave functions decrease the $B \rightarrow \pi$ transition form factors in the k_T factorization theorem, which is attributed to the suppression at the end point.

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A. Explicit expressions of the functions A_i and B_i

In this appendix we collect the explicit expressions of the functions A_i and B_i appearing in Eqs. (3.13) and (3.14):

$$A_2 = \ln \hat{\mu}_f - \frac{1}{2} \ln (\hat{\mu}_f^2 + \pi^2) , \quad A_3 = \phi_1 ,$$

$$A_4 = \frac{1}{2} \ln \left[\frac{\hat{\mu}_f^2}{(\hat{\mu}_f - t)^2 + \pi^2} \right] + \ln \left| \frac{\mu_f^2 - e^{2t} \Lambda_{\text{QCD}}^2}{\mu_f^2 - \Lambda_{\text{QCD}}^2} \right| , \quad A_5 = -\phi_2 - \pi \theta(t - \hat{\mu}_f) . \quad (\text{A.1})$$

$$B_2 = -\frac{\pi^2}{2} + \frac{1}{2} \hat{\mu}_f \ln (\hat{\mu}_f^2 + \pi^2) - \pi \phi_1 - \hat{\mu}_f \ln \hat{\mu}_f - \text{Li}_2 \left(-\frac{\mu_f}{\Lambda_{\text{QCD}}} \right)$$

$$+ \text{Re} \left[\text{Li}_2 \left(\frac{\mu_f}{\Lambda_{\text{QCD}}} \right) \right] - \text{Li}_2 \left(-\frac{\Lambda_{\text{QCD}}}{\mu_f} \right) + \text{Li}_2 \left(\frac{\Lambda_{\text{QCD}}}{\mu_f} \right) ,$$

$$B_3 = -\frac{\pi^2}{2} \ln (\hat{\mu}_f^2 + \pi^2) + \pi \left(\ln \frac{\mu_f^2}{\mu_f^2 - \Lambda_{\text{QCD}}^2} + \ln \hat{\mu}_f + 1 \right) - \phi_1 \hat{\mu}_f ,$$

$$B_4 = \frac{t^2 - \pi^2}{2} + \frac{\hat{\mu}_f - t}{2} \ln [(\hat{\mu}_f - t)^2 + \pi^2] - \pi \phi_2 - \hat{\mu}_f \ln \hat{\mu}_f$$

$$+ t \left(\ln \frac{\mu_f \Lambda_{\text{QCD}}}{\mu_f^2 - \Lambda_{\text{QCD}}^2} + \ln \hat{\mu}_f + 1 \right) - \text{Li}_2 \left(-\frac{\mu_f}{\Lambda_{\text{QCD}}} \right)$$

$$+ \text{Re} \left[\text{Li}_2 \left(\frac{\mu_f}{e^t \Lambda_{\text{QCD}}} \right) \right] - \text{Li}_2 \left(-\frac{e^t \Lambda_{\text{QCD}}}{\mu_f} \right) + \text{Li}_2 \left(\frac{\Lambda_{\text{QCD}}}{\mu_f} \right) ,$$

$$B_5 = -\pi t + \frac{\pi}{2} \ln [(\hat{\mu}_f - t)^2 + \pi^2] - \pi \left(\ln \frac{\mu_f \Lambda_{\text{QCD}}}{\mu_f^2 - \Lambda_{\text{QCD}}^2} + \ln \hat{\mu}_f + 1 \right)$$

$$+ \phi_2 (\hat{\mu}_f - t) - \pi (\hat{\mu}_f - t) \theta(\hat{\mu}_f - t) , \quad (\text{A.2})$$

with

$$\phi_1 = \arctan \frac{\pi}{\hat{\mu}_f} , \quad \phi_2 = \arctan \frac{\pi}{\hat{\mu}_f - t} \theta(\hat{\mu}_f - t) + \left[\pi + \arctan \frac{\pi}{\hat{\mu}_f - t} \right] \theta(t - \hat{\mu}_f) .$$

References

- [1] G.P. Lepage and S.J. Brodsky, Phys. Lett. B **87**, 359 (1979); Phys. Rev. D **22**, 2157 (1980).
- [2] A.V. Efremov and A.V. Radyushkin, Phys. Lett. B **94**, 245 (1980).
- [3] V.L. Chernyak, A.R. Zhitnitsky, and V.G. Serbo, JETP Lett. **26**, 594 (1977).
- [4] V.L. Chernyak and A.R. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 544 (1980); Phys. Rep. **112**, 173 (1984).
- [5] H-n. Li, Phys. Rev. D **64**, 014019 (2001); M. Nagashima and H-n. Li, Eur. Phys. J. C **40** (2005) 395.
- [6] A.G. Grozin and M. Neubert, Phys. Rev. D **55**, 272 (1997).
- [7] V.M. Braun and I.E. Filyanov, Z. Phys. C **48**, 239 (1990); P. Ball, JHEP **9901**, 010 (1999).

- [8] P. Ball, V.M. Braun, Y. Koike and K. Tanaka, Nucl. Phys. **B529**, 323 (1998).
- [9] H. Kawamura, J. Kodaira, C.F. Qiao, and K. Tanaka, Phys. Lett. B **523**, 111 (2001);
Erratum-ibid. 536, 344 (2002); Mod. Phys. Lett. A **18**, 799 (2003).
- [10] H.-n. Li and B. Melic, Eur. Phys. J. C **11**, 695 (1999).
- [11] Y.Y. Charng and H.-n. Li Phys. Rev. D **72**, 014003 (2005).
- [12] B.O. Lange and M. Neubert, Phys. Rev. Lett. **91**, 102001 (2003).
- [13] V.M. Braun, D.Yu. Ivanov, G.P. Korchemsky, Phys. Rev. D **69** (2004) 034014.
- [14] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999);
Nucl. Phys. **B591**, 313 (2000); Nucl. Phys. **B606**, 245 (2001).
- [15] P. Ball and E. Kou, JHEP **0304**, 029 (2003).
- [16] H. -n. Li and H. -S. Liao, Phys. Rev. D **70** (2004) 074030.
- [17] A. Szczepaniak, E.M. Henley, and S. Brodsky, Phys. Lett. B **243**, 287 (1990).
- [18] S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. B **242**, 97 (1990); Nucl. Phys. B **366**,
135 (1991).
- [19] J.C. Collins and R.K. Ellis, Nucl. Phys. B **360**, 3 (1991).
- [20] E.M. Levin, M.G. Ryskin, Yu.M. Shabelskii, and A.G. Shuvaev, Sov. J. Nucl. Phys. **53**, 657
(1991).
- [21] J. Botts and G. Sterman, Nucl. Phys. B **325**, 62 (1989).
- [22] H.-n. Li and G. Sterman, Nucl. Phys. B **381**, 129 (1992).
- [23] M. Nagashima and H-n. Li, Phys. Rev. D **67**, 034001 (2003).
- [24] H-n. Li, hep-ph/0304217.
- [25] T. Huang and Q.-X. Shen, Z. Phys. C **50**, 139 (1991); J.P. Ralston and B. Pire, Phys. Rev.
Lett. **65**, 2343 (1990); R. Jakob and P. Kroll, Phys. Lett. B **315**, 463 (1993); B **319**, 545
(1993)(E).
- [26] H-n. Li and H.L. Yu, Phys. Rev. Lett. **74**, 4388 (1995); Phys. Lett. B **353**, 301 (1995); Phys.
Rev. D **53**, 2480 (1996); H-n. Li, Chin. J. Phys. **34**, 1047 (1996).
- [27] C.H. Chang and H-n. Li, Phys. Rev. D **55**, 5577 (1997); T.W. Yeh and H-n. Li, Phys. Rev. D
56, 1615 (1997).
- [28] Y.Y. Keum, H-n. Li, and A.I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008
(2001); Y.Y. Keum and H-n. Li, Phys. Rev. **D63**, 074006 (2001).
- [29] C. D. Lü, K. Ukai, and M. Z. Yang, Phys. Rev. D **63**, 074009 (2001).
- [30] T. Kurimoto, H-n. Li, and A.I. Sanda, Phys. Rev D **65**, 014007 (2002); Phys. Rev. D **67**,
054028 (2003).
- [31] Y.Y. Keum, H-n. Li, and A.I. Sanda, AIP Conf. Proc. **618**, 229 (2002); Y.Y. Keum and A.I.
Sanda, Phys. Rev. D **67**, 054009 (2003).
- [32] H.-n. Li, ECONFC070512:011 (2007) [arXiv:0707.1294 [hep-ph]].

- [33] X. -G. He, T. Li, X. -Q. Li and Y. -M. Wang, Phys. Rev. D **75** (2007) 034011; P. Guo, H. -W. Ke, X. -Q. Li, C. -D. Lu and Y. -M. Wang, Phys. Rev. D **75** (2007) 054017; A. Ali, G. Kramer, Y. Li, C. -D. Lu, Y. -L. Shen, W. Wang and Y. -M. Wang, Phys. Rev. D **76** (2007) 074018; C. -D. Lu, Y. -M. Wang, H. Zou, A. Ali and G. Kramer, Phys. Rev. D **80** (2009) 034011.
- [34] J. C. Collins and D. E. Soper, Nucl. Phys. **B193**, 381 (1981); J. C. Collins, Adv. Ser. Direct. High Energy Phys. 5, 573 (1989).
- [35] J.C. Collins, Acta. Phys. Polon. B **34**, 3103 (2003).
- [36] H.-n. Li, Y.-L. Shen, and Y.-M. Wang, Phys. Rev. D **85**, 074004 (2012).
- [37] H.-n. Li, Y.-L. Shen, Y.-M. Wang, and H. Zou, Phys. Rev. D **83**, 054029 (2011).
- [38] J.C. Collins and F. Tkachov, Phys. Lett. B **294**, 403 (1992); J.C. Collins, PoS LC2008, 028 (2008) [arXiv:0808.2665 [hep-ph]].
- [39] J. -y. Chiu, A. Jain, D. Neill and I. Z. Rothstein, Phys.Rev.Lett. **108**, 151601 (2012); JHEP **1205** (2012) 084.
- [40] S. Fleming and O. Zhang, arXiv:1210.1508 [hep-ph].
- [41] T. Altinoluk, B. Pire, L. Szymanowski and S. Wallon, arXiv:1206.3115 [hep-ph]; T. Altinoluk, B. Pire, L. Szymanowski and S. Wallon, JHEP **1210** (2012) 049 [arXiv:1207.4609 [hep-ph]].
- [42] I.O. Cherednikov and N.G. Stefanis, Nucl. Phys. **B802**, 146 (2008).
- [43] H.-n. Li, Phys. Rev. D **55**, 105 (1997)
- [44] H.-n. Li, Phys. Lett. B **454**, 328 (1999).
- [45] E. Laenen, G. Sterman, and W. Vogelsang, Phys. Rev. Lett. **84**, 4296 (2000).
- [46] J. Debove, B. Fuks, and M. Klasen, Nucl. Phys. **B849**, 64 (2011).
- [47] J. P. Ma and Q. Wang, Phys. Lett. B **613**, 39 (2005); F. Feng, J. P. Ma and Q. Wang, arXiv:0901.2965 [hep-ph].
- [48] H.-n. Li, Phys. Rev. D **66**, 094010 (2002).
- [49] I. L. Solovtsov and D. V. Shirkov, Theor. Math. Phys. **120** (1999) 1220 [Teor. Mat. Fiz. **120** (1999) 482].
- [50] S. Wandzura and F. Wilczek, Phys. Lett. B **72** (1977) 195.
- [51] M. Beneke and T. Feldmann, Nucl. Phys. B **592** (2001) 3.
- [52] G. Bell and T. Feldmann, JHEP **0804** (2008) 061.